KNOWLEDGE SPACES AND HISTORICAL KNOWLEDGE IN PRACTICE

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ABSTRACT

One of the methods of historical knowledge assessment is the application of knowledge space theory. One of its most important elements is the construction of knowledge structure. This presupposes a prerequisite relation system between knowledge items, which can mostly be proved in a „logical way” in the case of exact sciences, however we cannot disregard the empirical justification of these relations in the case of history. The present study puts the aforementioned as its aim in one topic of Hungarian history, namely the life of Lajos Kossuth. The profundity of the questions equals the knowledge level expected at school-leaving exams. We made a questionnaire, which was filled in by 169 high-school and university students. We selected two groups of its questions, which can be related to more restricted topics. We examined the relations between them in each groups that we supposed to exist and based on this, we set up two prerequisite relation models.

Keywords: knowledge space, competence-performance based approach, historical knowledge

1. Introduction

The main motivation of our present study is the research of Hungarian cultural memory, and within this our long-term goal is acquiring knowledge on and comparing collective knowledge states in different age groups, as well as describing the structure and history of Hungarian sites of memory. The theoretical background of our study is offered by knowledge space theory (Doignon és Falmagne, 1999), which promises the following advantages for teachers in the exact definition of historical knowledge or memory related to a person or a group, as opposed to the traditional diagnostic and formative assessment of long standing in measuring students' knowledge level (Katona and Sallai, 2002): a) the assessment process is economical concerning the questions asked, and it is adaptive similar to oral assessment; b) it gives exact information on the individual's knowledge state, it does not regard providing a number (e.g. percentage or grade) enough; c) it also ensures the determination of knowledge structure representative of the group; d) it makes possible the comparability of individuals' or groups' knowledge states; and last e) the assessment is possible by means of a computer.

As a first step to achieve our goals, we tried to define the so-called surmise (or precedence relations or prerequisite relatin) required by knowledge space theory, and we presented the basic steps towards the procedure of computer-based assessment (Abari and Máth, 2010). In the present study, we conduct an empirical analysis of knowledge space theory – so far mostly used in mathematics and natural sciences – within the domain of history. The topic chosen is „Lajos Kossuth's walk of life", which is an inevitable site for the research of Hungarian memory.

Therefore, our study examines the question of how applicable knowledge space theory is in the assessment of the knowledge on history. First, we present the outlines of competence-based knowledge space theory, then we provide the description of the surmise relation we defined (the so-called depth relation). We also present the worksheet on history we prepared, whose response patterns based on the submissions of 169 students are compared to the theoretical model based on the worksheet questions. We outline the indexes on the goodness of fit regarding the theoretical models and the data, which show the degree of applicability of knowledge space theory.

2. Competence-based knowledge space theory

Knowledge space theory (KST) was formulated by Doignon and Falmagne with the application of the concepts of lattice theory in mathematics (Doignon and Falmagne, 1999). Their primary goal was to develop a formal system of tools which made possible the assessment of an individual's knowledge on a given domain in an adaptive way, with the help of a computer. A fundamental element of knowledge space theory is the so-called *precedence relation*, which sets up relations between questions (or problems, items) related to a given domain.

Example 1. Let our given domain be „Lajos Kossuth's walk of life". Let us suppose that the domain includes the following 3 questions only (the question codes are given in brackets, which will be presented consistently in what follows as well):

a) Is the concept of "Redemption" relavant in this domain? (SZ1_K4_BB)
b) Is the concept of "Serfdom" relavant in this domain? (SZ1_K5_P)
c) What was the meaning of "Redemption" mean? (SZ2_K6)

In this case, the domain is the set of problems $Q=\{a, b, c\}$. Formally, the surmise or precedence relation means the specification of a binary relation on set Q (its notation is: $\leq$). The precedence relation between problems a and b is noted as $(a, b) \in : \leq$, or the more readable $a: \leq b$ form is used. The interpretation of $a: \leq b$ is the following:

if a student succeeds solving problem „b", she or he will also be able to solve problem „a".

The following definition equals the above:

if a student has failed to solve problem „a", she or he will also fail problem „b".

It is evident that the following precedence relations hold between the 3 problems mentioned above: $a: \leq c$, $b: \leq c$. The relation $a: \leq c$ holds, as knowing "what Redemption is" (c), involves knowing „the relatedness of Redemption to the domain" (a), to put it in a different way, if we do not know „the relatedness of Redemption to the domain" (a), we cannot properly answer the question of „what Redemption is" (c) either. Similarly, $b: \leq c$ also holds, as the concept of Redemption requires the knowledge of Serfdom to be related to the domain. (Further detailed explanation on precedence relations can be found in chapter 4.2.) Let us note that in interpreting precedence relations, we may exchange the expression „succeeds solving a problem" to the expression with equal meaning „knows the answer", and by using the term „*at least as difficult"*, we may even reword the interpretation of the relation. The precedence relation between problems a and b holds ($a: \leq b$), if:

problem „b" is at least as difficult as problem „a".

The precedence relation of Example 1 is well representable by the so-called Hasse diagram (Diagram 1), which presents problems as points, and the precedence relations between them as ascending edges. (The exact definition of Hasse diagram can be found in Appendix 1.) As we do not direct the edges in our diagrams, we fix it that a directed edge runs from the lower point to the upper point in all cases. In line with this, a question at a lower level is precedence to all questions at higher levels which can be reached along the directed edges taking the lower-level question as the starting point.

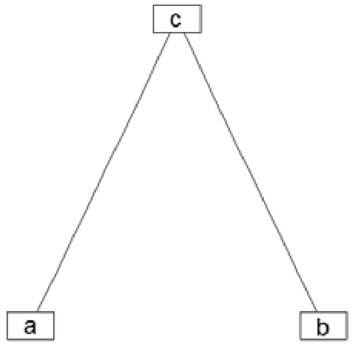

Diagram 1.
The Hasse diagram of the 3 problems in *Example 1.*

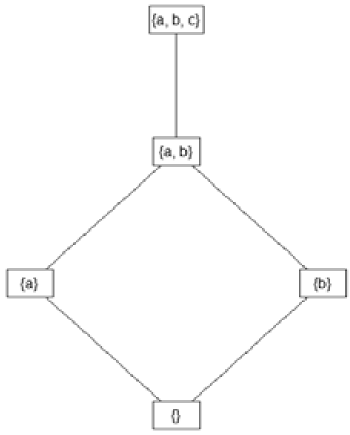

Diagram 2.
The Hasse diagram of the knowledge structure based on *Example 1.*

The result of the assessment process applying knowledge space theory is a so-called *knowledge state*, which comprises the problems the individual can answer correctly. Let K be the notation of a possible knowledge state. It is obvious that $K \subseteq Q$, that is, the given knowledge state is a subset of the problems. The possible knowledge states based on the precedence relations defined in the given domain, which are expected during the assessment procedure can easily be defined. Diagram 2 illustrates the possible knowledge states of Example 1 in a Hasse diagram. It can be noticed that set $\{a, c\} \subseteq Q$ is not present in Diagram 2, which is not by coincidence, as a knowledge state of this kind cannot occur based on the given precedence relations. That is, the correct solution of problem c results in the correct solution of problem b, so this in reality is knowledge state $\{a, b, c\}$, which is part of Diagram 2. This important characteristic feature, namely that the number of possible knowledge states is less than the number of subsets of Q is very useful in practical assessment. Taking into consideration the „restrictive" effect of all the precedence relations, we may call the set of possible knowledge states *knowledge structure*, and note it by $\mathcal{K}$. *Knowledge space* means pair $(Q, \mathcal{K})$, if $\mathcal{K}$ is closed under finite union.

In the analysis of historical memory sites, or even in school assessment when we determine an individual's knowledge state in a given domain, it is not certain that we expect the set of correctly answered questions as a result, but the hidden, cognitive, latent level is more important. Knowledge space theory basically focuses on the performance that can be observed during problem solution. One of its most successful extended branches that involves cognitive, latent structures as well is Competence Performance Approach (CPA), which was proposed by Klaus Korossy (Korossy, 1999). Korossy renamed knowledge space $(Q, \mathcal{K})$ known so far and which was based on problems as performance space and noted it (A, P), which comprises set A of problems and a performance structure P defined by the precedence relation between problems of set A. Latent structures explicitly appear in Korossy's model, as he also defines an (E, C) competence space, which is completely identical in structure with the already known $(Q, \mathcal{K})$ knowledge space. Through competence space (E, C), a set E of abstract cognitive abilities relevant for the domain was introduced, and its subsets were called competence states. These are, however, not directly observable, but are definable through the analysis of problems representing the domain. Competence structure C is determined by precedence relations defined on set E of elementary competences. The two constructions, namely performance space and competence space are formally identical (both have the already introduced concept of knowledge space $(Q, \mathcal{K})$ in their background), the only difference lies in their interpretation. Worded simply, it can be said that competence space has knowledge items that belong to the domain, whereas problem space has the questions that belong to the domain in its background.

Korossy links performance space and competence space with the so-called interpretation function: he assigns to each problem $a \in A$ the set of competence states in C, whose every element (competence state) makes it possible to solve problem a. After the provision of the interpretation function, the performance states are simply „readable" from the model: a set $Z \subseteq A$ of problems can be considered a performance state, if there exists a competence state $c \in$ C that Z contains exactly the problems which are solvable in c. Thus, the following need to be determined in Korossy's model: 1) set A of problems; 2) set E of elementary competences through the analysis of problems, 3) the precedence relations between elementary competences and the related competence structure C; 4) an interpretation function for linking performance space and competence space. Determining the performance structure which is the starting point of the adaptive assessment procedure is done automatically.

Competence based knowledge spaces have only been outlined in the present chapter. The detailed description of knowledge space theory can be found in two books by the theory's creators (Doignon and Falmagne, 1999, 2010), and it is also possible to have access to the briefer summaries of the topic in Hungarian (Tóth, 2005, Abari and Máth, 2010). More details on the competence-based extension can be found in Korossy (1999), and in Abari and Máth (2010) in Hungarian.

3. WORKSHEET ON HISTORY

According to what has been proposed in the above chapter, the determination of precedence relations between elementary competences is of fundamental importance in assessment procedures based on competence-based knowledge space. Therefore, it is our significant objective to get hold of knowledge expected at the school leaving exam with questions that are constructed along a certain hierarchy. By using the so-called *depth relations* (Abari and Máth, 2010), we distinguished four levels in the hierarchy, with four different questions:

Level 1: Are certain things (individuals, events, places, concepts, etc.) relevant from the point of view of the given domain?

Level 2: The meaning of the things mentioned at Level 1 are asked about, and fundamental things concerning them.

Level 3: The correlation between things mentioned at Level 2 are asked about, and correlations that concern the things mentioned at previous levels.

Level 4: We ask questions concerning deeper correlations related to the things mentioned at the previous three levels, where the alternatives proposed are more complicated and contain several sentences.

It is a fundamental question how depth relations operate in practice.

In order to analyse precedence relations, we made a worksheet with the domain of „Lajos Kossuth's walk of life”. We divided the questions of the worksheet into four stages, based on their difficulty, and we paid attention to the depth relation. We asked questions about the same knowledge several times, in several forms, as it is exactly what the depth relation requires. Thus, „easy” questions were posed first, then the ones requiring deeper and deeper knowledge. We tried to achieve to have a correspondence between the levels of the depth relation and the stages.

The grouping of exercises by stages and their form is presented in Table 1. We deemed it important to avoid human intermediation in the evaluation of questions, as this is the only way to achieve the computer-based assessment procedure that is fast enough, and therefore can be widely used. Thus, all the questions are closed-ended, yes or no and multiple choice questions, with ample response alternatives, whose number is enough to achieve that those who bet should not be able to find the right answer very easily, but it should not be too difficult either to find possible and believable answers (Katona and Sallai, 2002).

Table 1. Grouping of the questions of the history worksheet

Stage	Number of Questions Total: 181	Exercise form	Correct/total number of answers
1.	135 questions (in 5 subdomains: places, people, documents, events, groups)	Yes or no	Different in the 5 subdomains: 10/22; 21/39; 10/20; 13/34; 9/20
2.	24	Multiple choice	1/4
3.	18	Multiple choice	1/4
4.	4	Multiple choice	1/4

The worksheets were filled in by 183 individuals in 4 high schools and at a university in May 2011. The completion took place during regular classes, with the supervision of the teachers of history, who had been previously provided with a written guide to assist them. Students had 20-24 minutes to fill in the forms, which meant 5-6 minutes on each stage. It was observed that many overstepped the time limit, though not significantly, but they needed approximately 30 minutes to fill in the forms with no rush. The completion of the worksheet took place stage by stage, as first they answered questions of stage 1, then they turned to the questions of stage 2, but they could no longer modify their answers at stage 1, and so on, the questions of the previous stages were no longer accessible. This was an important criterion in the completion process with questions gradually becoming more difficult by every stage.

When analysing the data, we omitted the data of those who did not respond to more than two questions, thus of the starting data sample we omitted the data of 14 students. This way, the database used for data analysis contains responses by 169 individuals. The sample comprises responses by 138 high-school students and 31 university students. Table 2 shows information on respondents' mean age, their gender distribution, as well as their attitude towards history.

Table 2. Some statistical data on respondents.

	Number	Mean Age (year)	Sex*		Attitude towards History**			
			Male	Female	1	2	3	4
high school student	138	17.42	34%	64%	19%	51%	27%	3%
student	31	21.19	16%	84%	10%	35%	48%	6%
Total	**169**	**18.11**	**31%**	**68%**	**17%**	**49%**	**31%**	**4%**

* There were respondents who did not indicate their sex
** 1 – I do not like it; 2 – It is OK; 3 – I like it; 4 – It is my favourite subject

When selecting the questions of the worksheet, we tried to cover the material on Kossuth in Miklós Száray's high-school history book (Száray, 2009) as much as possible. In compiling the list of relevant people and places, we also used the essay by Z. Király (Király, 2010), which gives a review on the people and places occurring most often in history books used in secondary schools. The wording and checking of the questionnaire were assisted by Ferenc Velkey, associate professor of the Institute of History.

4. Determining the performance structure

One of the most important objectives of our present study is to examine how much and in what way knowledge space theory can be applied in mapping historical knowledge. The basis of the adaptive assessment procedure is performance structure (that is, the traditional knowledge space), and we show an example to define this in the present chapter. First, we present the application of the depth relation with the so-called component analysis method, which leads to a set of elementary competences and competence states, and indirectly it results in performance structure. Following this, we turn to the statistical analysis of connections between questions based on logical relations expected by experts. We selected two groups of questions, which can be related to more restricted topics. We examine the connections between them in each groups that we supposed to exist and based on this, we set up two prerequisite relation models.

4.1. Component analysis

The component-based establishment of surmise relations was introduced by Albert and Held (1999). With the analysis of problems of a given Q domain, we break up the problems into components, and create a set of components, *Comp* of them. It is an important condition that some form of order (quasi order, linear order) should prevail between elements of *Comp*. As in this case, the order between components provides the hierarchy of problems made up of components. Appendix 1 offers more information on the different cases of order.

Let us apply the method of component analysis to elementary competences. (In this case, the adjective „elementary" becomes less important, as we try to analyse and break up competences.) The depth relation between elementary competences is extremely appropriate for this purpose.

Example 2. Let us suppose that elementary competences related to domain *Q* can be described by a single attribute, their depth level. Let us note it by *Sz*, and on the basis of the above, it is obvious that *Sz*={1, 2, 3, 4}, where numbers symbolize depth levels. There is linear order between the 4 levels (Diagram 3). If the set of components was identical with set S*z*, there would exist only 5 elementary competences, in connection with the precedence relation illustrated by Diagram 4.

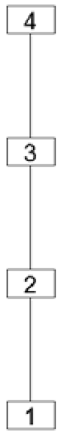

Diagram 3.
The Hasse diagram of the linear order between the depth levels, based on *Example 2.*

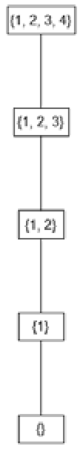

Diagram 4.
The set of elementary competences, if there is linear order between the elements of the set of components (based on *Example 2*).

Naturally, an elementary competence is not only characterised by its depth level, but also by what the knowledge level concerns. That is, what is it we know at level 1, level 2, etc. So we introduce set *D* of things, which contains the important concepts, individuals, groups, events, etc. of the given domain.

Example 3. Let the set of things in our domain be the following set: *D*={Sz, e, o}, where the following abbreviation is used: **Sz**="István Széchenyi", **e**="Union of Interests", and **o**="Redemption". Let us consider the set of things its power set, and let us take away the empty set: $2^{D+}=2^{D}\setminus\varnothing$={{Sz}, {e}, {o}, {Sz,e}, {Sz,o}, {e,o}, {Sz,e,o}}. Let us consider the inclusion relation over set 2^{D+}, which is partial order and so quasi order as well.

On the basis of the above, our elementary competences have two attributes:

- *Sz*: the level of depth relation, which is ordered linearly.
- 2^{D+}: the power set of things without the empty set, which is partially ordered with the inclusion relation.

Albert and Held (1999) showed how knowledge space can be established in the case of two (or more) attributes. This method can also be used in the case of elementary competences. Let us form the Cartesian product of the two sets, which results in the set of elementary competences: $E=Sz\times 2^{D+}=\{(1,\{\mathrm{Sz}\}), (1,\{\mathrm{e}\}), \ldots, (4,\{\mathrm{Sz,e,o}\})\}$. Set E of elementary competences has 28 elements. The prerequisite relation between them comes about on the basis of the rule of coordinatewise order and along the orderliness of set *Sz* and 2^{D+}. In general, if ($Comp_1$, R_1) and ($Comp_2$, R_2) are partially ordered component sets, then *a b holds* (that is, *a* prerequisites for *b*), if $a, b \in Comp_1 \times Comp_2$ and $a = (a_1, a_2)$, $b = (b_1, b_2)$ holds, $a_1R_1b_1$ and $a_2R_2b_2$ are fulfilled. In Example 3, $Comp_1=Sz$ and $Comp_2=2^{D+}$, furthermore, R_1 is the linear order between levels, R_2 is the inclusion relation between elements of 2^{D+}. Diagram 5 shows set E of elementary competences comprising 28 members, and the precedence relations defined according to the rule of coordinatewise order. The labels of nodes denote elementary competences in a simplified way. Label „1-Sz", for example, denotes elementary competence (1, {Sz}), while „3-Sze" denotes element (3,{Sz,e}) in *E.* The meaning of „1-Sz" is: "s/he knows that István Széchenyi belongs to the domain", whereas the meaning of "3-Sze" is: „s/he is familiar with the relation between István Széchenyi and the Union of interests".

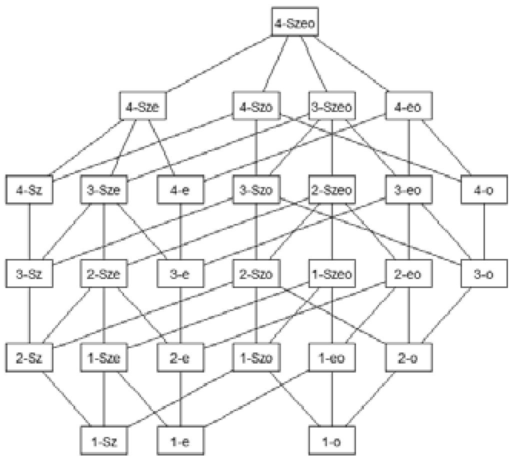

Diagram 5.
The Hasse diagram of elementary competences.

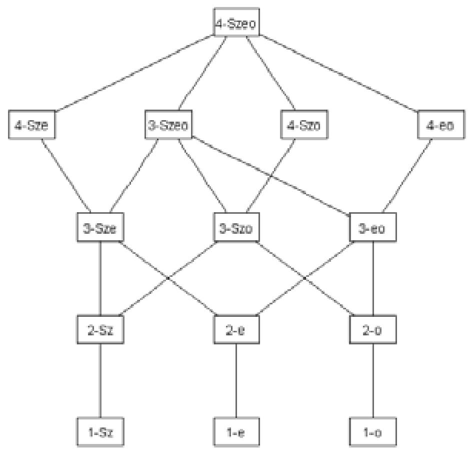

Diagram 6.
The Hasse diagram of the restricted set of elementary competences.

The two elementary competence attributes defined above in the domain of history reflect a relatively simple knowledge structure. The attribute of levels (*Sz*) shows the depth of knowledge, which can be extended to further levels at will, or restricted in certain cases. The power set of things (2^{D+}) grasps the subject of knowledge, which can be one-element sets, for example {István Széchenyi}, or multi-element sets {István Széchenyi, Union of interests}. The definition of set *D* of things makes the use of this tool very free, and it is only up to us what we include or leave out of the system. These are typically the one-element sets of 2^{D+} that are important at the first two levels concerning elementary competences: the given thing belongs to the subject (level 1), or the person knows its meaning (level 2). At higher levels, the multi-element sets of 2^{D+} play an important role: the person knows the relation between István Széchenyi and the Union of interests. According to the above, we may restrict set *E* of elementary competences, we may omit the elementary competences of the first two levels that refer to multi-element things (e.g.: „1-eo" and „1-Sze") and the ones of the higher two levels that refer to one-element things (e.g.: „3-e” and „3-o”). Diagram 6 shows the restricted set *E* of elementary competences, which thus contains only 14 elementary competences.

In order to create performance space, we need to assign to our questions the competence states where the given problem can be solved successfully (interpretation function). Let us note that the original competence structure *C* (based on Diagram 5) includes 2603 competence states, while the restricted one (based on Diagram 6) only 74. The number of assignments can be further decreased in practice with the use of the so-called base, which has now 14 elements. Details on determining performance space can be found in Korossy, 1999, and Abari and Máth, 2010 also present it through an example.

Let us note that the implementation of the above procedure runs into difficulties when the element number of the set of attributes is increased. If the set of things is extended by one single element (e.g.: the concept of Serfdom), the restricted set of competence states will have 30 elements, and the competence space will have 3454 elements. As the running time of our experiments to determine the base for creating performance space can be measured in days (*R* program with *kst* package, 3.1 GHz CPU, Windows 7) – with a personal computer that can be considered up-to-date today –, we shall not consider the further examination of the very promising method based on component analysis in our present study.

4.2. Statistical analysis of the questionnaire survey

As we mentioned in chapter 3, we had a questionnaire of four sections with 181 items altogether – 135 binary items in the first section, 24, 18 and 4 multiple choice items with four alternatives in the remaining sections. The answers of 169 persons were coded into a binary data matrix of 169 rows and 181 columns. Each row is a so called response *pattern* of zeros and ones, where zero means the wrong answer, and one means the right one.

In this section we examine some pairs of questions, between which an expert would find prerequisite relation (see chapter 2.) and we investigate whether our data confirm this relation. More precisely, in case of a pair of questions, we examine what is the consequence of the right answer to the first question for the correctness of the second answer. The knowledge space theory assumes that the correct answer to the first question is necessary for the right answer to the second one, that is, the wrong answer to the first question implies the wrong answer to the second one.

For example, if the first question is related to “Széchenyi”, while the second one is about the “essence of the Kossuth – Széchenyi debate”, we may assume that those who don’t mark Széchenyi, as a relevant person, know nothing about the debate.

It is possible at the same time, that if the topic is Kossuth, Széchenyi doesn’t cross one's mind, but the appropriate question dredges the right answer. In this case the strength of relation between the two questions will decrease, the wrong answer to the first question only decrease the chance of a right answer to the second question, but it doesn’t exclude it.

Another factor, which may have an opposite impact is that students usually read more pages or a whole chapter at once and this establishes relations between the answers to questions close to each other in the book but distant from each other logically.

Consequently, when examining a statistically significant relation between two items, it is not easy to decide whether there is a prerequisite relation between them or not. The aim of this paper – inter alia – is just to answer this question.

To do this we selected two sets of questions, each of them related to the same topic, and on the basis of their logical connections we set up two systems of relations, which can be tested within the knowledge space theory.

4.2.1 Examining question pairs of level 1 and 2

When examining the pairs of questions, we assumed the strongest prerequisite relations between the questions of level 1 and 2. It means that if someone doesn't mark a concept on level 1, he or she doesn't know the meaning of this concept and won't be able to answer any question of level 2 based on or connected with the concept. We also assumed, slightly pushing the boundaries of this hypothesis, that a mistake on level 1 results in guessing in case of other questions related to that concept.

In Table 3 we listed a few pairs of questions from level 1 and 2 with the most important descriptive values of their relations in order to demonstrate the difficulties in applicability of knowledge spaces. Table 3 summarises the following information:

Column 1: the first question
Column 2: proportion of correct answers to the first question
Column 3: the second question
Column 4: significance value, testing independence between the answers to the pair of questions
Column 5: proportion of correct answers to the second question in case of wrong and right answers to the first question, respectively.

In the second raw it can be seen that 26% of those who failed to mark "Union of Interests" could define this concept, while among the others this proportion was 49%. The independence of the answers to the pair of questions can be rejected because of the small significance. 26% is close to the proportion of success in case of guessing.

For those who are guessing, the possibility that their mean score exceeds the limit of , is 0.05, where n is the number of those who failed to mark the appropriate first level concept. This value is between 32 and 35%, depending on n occurring in our sample.

We may assume that for those who don't mark the concept in the first column the proportion of correct answers will be fewer than 35% in case of the covering question. We have multiple choice questions with 4 alternatives, and it is difficult to achieve that for a student who is uninformed about the given question all the alternatives seem equally possible. This is a likely explanation why the proportion of the right answers may exceed the level of 35%. If this proportion is much higher for the second (covering) question than for the first (prerequisite) question, it doesn't fit into the logic of knowledge space theory. This is the case for the last four questions. Let we see the alternatives and their frequencies.

Table 3. Testing independence of question pairs

1 level: Is that concept relevant?	**Prop. of right answers**	**Higher level question**	**Sign.**	**Prop. of right answers**	
National Defense Committee	45%	Who was the head of the National Defense Committee? (SZ2_K15)	0.045	23%	37%
Union of Interests	37%	What was the Union of Interests in the 1840s? (SZ2_K5)	0.003	26%	49%
serfdom	32%	What was the Union of Interests in the 1840s? (SZ2_K5)	0.001	26%	54%
National Defense Committee	45%	What was the meaning of National Defense Committee? (SZ2_K14)	0.080	33%	46%
Dietal Reports	70%	What was the meaning of Dietal Reports? (SZ2_K3)	0.007	35%	58%
Cassandra-letter	34%	What was the meaning of Cassandra-letter ? (SZ2_K21)	0.001	36%	63%
Széchenyi	78%	What was the essence of the Kossuth-Széchenyi debate? (SZ3_K7)	<0.001	36%	70%
National Protectionist Association	47%	What was the National Protectionist Association? (SZ2_K2)	<0.001	37%	72%
National Defense Committee	45%	How did National Defense Committee take over the power? (SZ2_K17)	0.011	41%	60%
Opposition Circle	40%	What was the meaning of Opposition Circle? (SZ3_K7)	0.200	42%	52%
Redemption	32%	What was the meaning of Redemption? (SZ2_K6)	0.040	55%	74%
Serfdom	32%	What was the meaning of Redemption? (SZ2_K6)	0.002	50%	83%
Address to the Crown	49%	What was the Address to the Crown? (SZ2_K11)	0.380	66%	72%
Jelasics	25%	Who was Jelasics? (SZ2_K13)	0.490	79%	74%

What is the meaning of Redemption?
(A) *Exemption for the nobility from inheritance tax.*
(B) *The plot of land becomes the serfs' own property and they are also exempted from all the other services toward their landlords*
(C) *The serfs' right to choose their landowners*
(D) *The serfs moving into towns may free themselves from corvées (labour services) by paying redemption fee.*

Alternatives to Question SZ2_K6.

Table 4. Crosstabulation of questions SZ1_K4_BB and SZ2_K6

		What was the meaning of Redemption? (SZ2_K6)				Total
		A	B	C	D	
Redemption	not marked	21 18.6%	62 54.9%	6 5.3%	24 21.2%	113 100.0%
	marked	3 5.4%	40 71.4%	1 1.8%	12 21.4%	56 100.0%
Total		24 14.2%	102 60.4%	7 4.1%	36 21.3%	169 100.0%

In the case of question "What was the meaning of Redemption?" there was an alternative chosen by a few students only. Almost nobody thought that alternative (C) was a lifelike answer. In this case this wrong option could explain the fact that for even those who failed to mark the concept of Redemption the proportion of correct answer was about 55%. For the question about the relation of "Serfdom" (SZ1_K5_P) and "Redemption" the situation was the same. Here the problem was that half of the students who failed to mark "Serfdom" gave a correct answer to the second (covering) question.

Let we examine this problem for the penultimate pair of questions in *Table 3.*

What was the Address to the Crown?
(A) *The regulation of the Dieta's agenda*
(B) *Proposal for introducing protective tariffs*
(C) *It proposed that the Hungarian soldiers stationing in foreign countries should be sent home*
(D) *It demanded obligatory redemption, burden sharing and independent Hungarian Government of the Emperor*

Alternatives to Question SZ2_K11

Table 5. Crosstabulation of questions SZ1_K3_T and SZ2_K11

		What was the Address to the Crown? (SZ2_K11)				Total
		A	B	C	D	
Address to the Crown	not marked	3 3.4%	6 6.7%	21 23.6%	59 66.3%	89 100.0%
	marked	6 7.5%	4 5.0%	12 15.0%	58 72.5%	80 100.0%
Total		9 5.3%	10 5.9%	33 19.5%	117 69.2%	169 100.0%

From the alternatives to the question "What was the Address to the Crown?" almost nobody choose the first two regardless of their answer to the first question concerning the relevance of "the Address to the Crown". So the "regulation of the Dieta's agenda" and the "introduction of protective tariffs" wasn't credible at all.

This is the last pair of questions in *Table 3.* SZ2_K13

Who was Jelasics?
(A) *Imperial general of Croatian ancestry*
(B) *Serbian rebel*
(C) *Croatian border guard colonel*
(D) *Slovenian colonel*

Alternatives to Question SZ1_K2_FF

Table 6. Crosstabulation of questions SZ1_K2_FF and SZ2_K13

		Who was Jelasics? (SZ2_K13)				Total
		A	B	C	D	
Jelacics	not marked	100 79.4%	9 7.1%	8 6.3%	9 7.1%	126 100.0%
	marked	32 74.4%	3 7.0%	8 18.6 %	0 .0%	43 100.0%
Total		132 78.1%	12 7.1%	16 9.5%	9 5.3%	169 100.0%

In the case of question „Who was Jelasics" we also had two alternatives that proved unbelievable, but the odd thing to note is that among those who marked Jelasics's name the proportion of the good answer is lower and the proportion of the third, wrong answer is higher. Perhaps the reason is that "General Jelasics" is a well-known name and only one alternative contained the title General. But, if someone marked Jelasics, he or she was likely to know Jelasics's nationality, which was a true piece of knowledge in the third alternative.

4.2.2. Examining pairs of questions of higher levels

Now let we see examples for connections between some higher level questions. Here we also can observe cases when that the prerequisite question proved to be more difficult than the covering one.

The concept of "Union of Interests" was familiar only to 35% of the students, but among these students the proportion of correct answers to the question "What was the opinion of Kossuth about the Union of Interests ?" was more than 50%.

What was the Union of Interests in the 1840s?
(A) Harmonizing the interests of Austria and Hungary
(B) *Harmonizing the interests of Hungarians and the other nationalities*
(C) *The concurrent respect of the noblemen's and the serfs' interests*
(D) *It was in the unified interests of the great powers not to let Hungary remain independent*

Alternatives to Question SZ2_K5

What was Kossuth's opinion about the Union of Interests? (A) *Hungary's independence cannot be achieved without this* (B) *This makes easier to achieve independence* (C) *It is easier to achieve independence without this* (D) *It increases the tension unnecessarily*

Alternatives to Question SZ3_K3

Table 7. Testing independence of question pairs

Prerequisite question	**Prop. of write answers**	**Covering question**	**Sign.**	**Prop. of write answers**	
What was the meaning of Redemption? (SZ2_K6)	60%	What was the Union of Interests in the 1840s? (SZ2_K5)	0.14	28%	39%
What was the Union of Interests in the 1840s? (SZ2_K5)	35%	What was the opinion of Kossuth about the Union of Interests ? (SZ3_K3)	0.01	50%	71%
What was the Union of Interests in the 1840s? (SZ2_K5)	35%	What is the connection between the Union of Interests and Redemption? (SZ3_K4)	0.01	18%	36%
What was the Union of Interests in the 1840s? (SZ2_K5)	35%	What was the essence of the Kossuth-Széchenyi debate? (SZ3_K7)	0.26	59%	68%
What is the connection between the Union of Interests and Redemption? (SZ3_K4)	24%	Which is the correct summary of the Kossuth-Széchenyi debate? (SZ4_K2)	0.09	30%	44%
What was the essence of the Kossuth-Széchenyi debate? (SZ3_K7)	62%	Which is the correct summary of the Kossuth-Széchenyi debate? (SZ4_K2)	0.46	30%	35%

We may presuppose that those who are not familiar with the concept of „Union of interests" also have no idea about Kossuth's opinion. The problem is that the last alternative, the statement "It increases the tension unnecessarily" did not proved to be credible therefore almost nobody choose it. It is a positive concept, which is not likely to increase the tension. The truth is that for a number of noblemen it did increase the tension but according to Kossuth this certainly was not unnecessary.

What was the essence of the Kossuth-Széchenyi debate? (A) *How to time the claim for Hungary's independence?* (B) *Is it possible to establish Hungarian industry without protective tariffs?* (C) *Is it advisable to speed it up the reform processes if this may lead to more severe conflicts between Austria and Hungary?* (D) *Is it possible to involve the serfs in the changes without risking the noblemen's rights?*

Alternatives to Question SZ3_K7

Table 8. Crosstabulation of questions SZ2_K6 and SZ4_K2

		What was the essence of the Kossuth-Széchenyi debate? (SZ4_K2)				Total
		A	B	C	D	
What is the meaning of Redemption?	A	10 20.4%	1 2.0%	29 59.2%	9 18.4%	49 100.0%
	B	9 18.4%	6 12.2%	28 57.1 %	6 12.2%	49 100.0%
	C	2 3.4%	2 3.4%	40 69.0 %	14 24.1%	58 100.0%
	D	2 18.2%	1 9.1%	7 63.6 %	1 9.1%	11 100.0%
Total		23 13.8%	10 6.0%	104 62.3%	30 18.0%	169 100.0%

The knowledge about the „Union of interests" and the "Kossuth-Széchenyi debate" may be related. The last alternative to the second question was connected with the „Union of interests" but it was a wrong alternative.

From the cross tabulation of the answers we can see that apart from the right answer this wrong alternative was almost the only one that was selected, moreover –compared to the other alternatives– in a bigger proportion. This is another example for a special kind of mistakes, when a piece of knowledge may increase the chance of a wrong answer merely because that answer contains the familiar piece of knowledge.

At first glance, the last question pair request for the same information, but this it is not the case. Though both of them are concerned with the "Kossuth - Széchenyi debate", the alternatives and the complexity of the answers are quite different. In the first item the alternatives differ greatly, which makes relatively easy to choose the right one, while in the second one the alternatives are similar and much more detailed.

Which is the correct summary of the Kossuth-Széchenyi debate?

(A) <u>*Széchenyi and Kossuth shared opinions on civil transformation and Hungary's autonomy but held divergent views about how to achieve these goals. Széchenyi was afraid of the quick growth of social contrasts and the conflict with the Habsburgs. Because of this Széchenyi openly attacked Kossuth, the editor-in-chief of the newspaper 'Pesti Hírlap' for his radical articles.*</u>

(B) *Széchenyi and Kossuth shared opinions on civil transformation, but Széchenyi did not accept the idea of Hungary's autonomy, while Kossuth attached the greatest importance to it. In this sense Széchenyi was a 'bridge man' because he wanted to keep Hungary and Austria together by any means.*

(C) *Széchenyi was a wealthy aristocrat and Kossuth belonged to the lower nobility with no land, so they saw the question of the civil transformation very differently. Széchenyi hurried the emancipation of serfs because wealthy landowners could afford to hire wage labourers using machines. Kossuth took a more cautious approach because the lesser nobles lacking capital couldn't go without the serfs' work.*

(D) *Széchenyi was born into a family traditionally loyal to the Habsburg dynasty. He had close connections with the Court in Vienna and as an officer swore allegiance to the Emperor. For emotional reasons Széchenyi did not consider acceptable the open collision with Habsburgs but because of Kossuth political activity it seemed more and more inevitable for him. Although Lajos Kossuth considered Count Széchenyi "the greatest. Hungarian", he added that Széchenyi should not be the greatest Austrian at the same time.*

Alternatives to Question SZ4_K2

Examining the answers to this pair of questions we didn't find significant connection. The question of level 4 proved to be very difficult, the proportion of correct answers didn't exceed 35%. Though this fact was surprising for us, it doesn't contradict the knowledge space theory. We may also note that the answer to this question is significantly related to the one about the connection between the "Union of Interests" and "Redemption". Both of them differ from the most questions because they require some intelligence.

Summarizing the results of the analysis above, it can be said, that the connection between the questions proved to be significant in the great majority of the cases. This means that the correct answer to the prerequisite question significantly increased the possibility of correct answer to the covering question, and it points in the direction of applicability of the knowledge space theory. There are some cases, however, when either this was not true or the wrong answer to the prerequisite question was followed by a high proportion of good answers to the covering question. The latter may limit the applicability of the knowledge space theory.

We think that this problem is connected with the fact that when answering a multiple choice question students may not only guess but also rely on some other pieces of information and heuristics which are not related the goodness of the answer to the prerequisite question:

- We presented examples for the fact that superficial knowledge increases the chance of choosing a familiar answer even if it is wrong.
- It also occurred that the correct answers to two very similar questions were not related to each other because the wrong alternatives of the items covered different areas.

All this means that in case of multiple choice tests the questions and the correct alternatives by themselves do not provide enough information to define the connections between the answers, because the wrong alternatives may distort the assumed structure significantly.

4.2.3. The suggested performance structure: the theoretic model

In this chapter we show two possible performance structures, which consist of 10 and 11 questions. The first topic is concerned with the "Union of interests", its connection with "Kossuth", and the "Széchenyi-Kossuth debate", which occurs mainly in the wrong alternatives. The second topic covers "Kossuth's opinion about Conciliation", and his role played in the military defeat.

The questions of the first topic can be seen in Table 9. We defined a hierarchical model of prerequisite relations in order to test them within the framework of the knowledge space theory. The Hasse diagram of this hierarchy can be seen on Diagram 7. This is the first theoretical model. The lines of this diagram are labelled with two values of percent. These are identical to those in the last two columns of Table 3 and 7. These are the proportions of good answers to the covering questions among those who gave wrong or good answers to its prerequisite question. The value in the square brackets of a question shows the overall proportion of good answers.

Table 9. The 11 questions of the first topic.

	The code of question	The question in the test
1.	SZ1_K2_P	Is István Széchenyi relevant in this domain?
2.	SZ1_K4_D	Is the „Union of interests" relevant in this domain?
3.	SZ1_K4_BB	Is the „ Redemption" relevant in this domain?
4.	SZ1_K5_P	Is the „ Serfdom" relevant in this domain?
5.	SZ2_K5	What was the Union of Interests in the 1840s?
6.	SZ2_K6	What was the meaning of „Redemption"?
7.	SZ3_K3	What was the opinion of Kossuth about the „Union of interests"?
8.	SZ3_K4	What is the connection between the Union of Interests and Redemption?
9.	SZ3_K6	How did the famous Széchenyi-Kossuth debate start?
10.	SZ3_K7	What was the essence of the Kossuth-Széchenyi debate?
11.	SZ4_K2	Which is the correct summary of the Kossuth-Széchenyi debate?

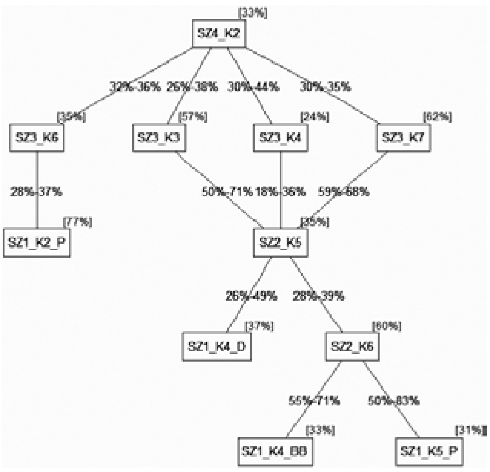

Diagram 7. The Hasse diagram of the first theoretical model representing 11 questions and their prerequisite relations

– The lines mean relations in which according to our hypothesis, the lower questions are the prerequisite of the upper ones. For example, in case of question SZ1_K4_BB ("Is the Redemption relevant?") and question SZ2_K6 ("What was the meaning of Redemption?") we assume that a correct answer to the first question is necessary to answer the second one. The summary of the previous model is he following.

- Marking „Redemption” (SZ1_K4_BB) and marking „Serfdom” is necessary to answer the question „What was the meaning of Redemption?
- Answering the question „What was the meaning of Redemption?” and marking the “Union of interests” necessary to answer the question “What was the Union of Interests in the 1840s?”
- Answering the question „What was the meaning of Union of interests?” (SZ2_K5) is necessary to answer the following questions.
  • “What was the opinion of Kossuth from the Union of interests?” (SZ3_K3)
  • „What is the connection between the Union of Interests and Redemption?” (SZ3_K4)
  • “What was the essence of the Kossuth-Széchenyi debate?” (SZ3_K7).
- Marking Széchenyi (SZ1_K2_P) is necessary to answer the question “How did the famous Széchenyi-Kossuth debate start?” (SZ3_K6).
- To answer the questions SZ3_K3, SZ3_K4, SZ3_K6, SZ3_K7 explained above are necessary to answer the question „Which is the correct summary of the Kossuth-Széchenyi debate?”

Let us have an example for the meaning of the per cent values on Diagram 7. For the SZ1_K4_BB question, the 33% in the upper right corner means the overall proportion of correct answers. The 55% - 71% pair stands for the same in the case of SZ2_K6 in the groups of wrong and right answers, respectively. These values imply two things: question SZ1_K4_BB proved to be much more difficult than the question SZ2_K6 and the right answer to the question SZ1_K4_BB significantly increases the chance of good answer to the question SZ2_K6.

In order to verify this hierarchy in the framework of the knowledge space theory it should be true for the most pairs of question that: (a) Those who give a wrong answer to a prerequisite question do the same in the case of the covering one (that is the proportion of good answers shouldn’t exceed 35%). (b) For the others the covering question is not easier, than the prerequisite question.

The questions of the second topic can be seen in *Table 10.*

The theoretic model 2 made of the above questions can be seen on Diagram 8. It is important to mention that in case of the last question (SZ4_K4) we subsequently decided to accept the second alternative too, because it is very close to the first one. It means that in this case for those who are guessing the chance of a good answer is not 25% but 50%

In connection with the previous topic’s figure we explained the meaning of lines and values, so we don’t repeat it again. That is why the analysis of the second model is significantly shorter.

One can see that relations of question pairs are consistent with the knowledge space theory. There is only one exception, the pair of (SZ1_K2_HH – “Is Görgey relevant ...” and SZ3_K15 – “Who was the person, against whom the other three …”) . Independently from marking “Görgey” the proportion of those choosing him as a correct answer for the second question was almost 50%. This value is too high compared to the ideal value of 25% for those who missed to mark Görgey.

An other interesting feature of this question pair is that a correct answer to the prerequisite question doesn’t increase the proportion of good answers to the covering question. The same

Table 10. The 10 questions of the second topic.

	The code of question	The question in the test
1.	SZ1_K2_F	Is „Ferenc Deák" relevant in this domain?
2.	SZ1_K2_P	Is „István Széchenyi" relevant in this domain?
3.	SZ1_K2_HH	Is „Artúr Görgey" relevant in this domain?
4.	SZ1_K3_B	Is „Casndra - letter" relevant in this domain?
5.	SZ1_K5_K	Is „Polish emigrants" relevant in this domain?
6.	SZ2_K21	What was the meaning of Cassandra – letter?
7.	SZ3_K15	Against whom did the other three persons come to an agreement during the Hungarian War of Independence?
8.	SZ3_K17	In the course of the fights against the Austrian – Russian army what was Kossuth's attitude like, which could be criticized subsequently?
9.	SZ3_K18	What was Kossuth's opinion about the „Union of interests"?
10.	SZ4_K4	Interpret the following statement told by Kossuth after the Hungarian War of Independence. "Poor of our country has been defeated not by the strength of our enemy, but by betrayal and villainy …"

occurs in the pairs (SZ3_K15 - SZ4_K4) and (SZ3_K17 - SZ4_K4). It is the case in the last two pairs despite the fact that all the questions are centred around the "Kossuth – Görgey conflict". It is noteworthy that the situation was similar in the case of the question pair (SZ3_K7 - SZ4_K2) centred around the „Kossuth – Széchenyi conflict".

All this doesn't contradict the knowledge space theory. It rather means that the questions of level 4 require, among other things, a competency that most of students don't pessess. This probably is the ability of understanding and comparing longer alternatives.

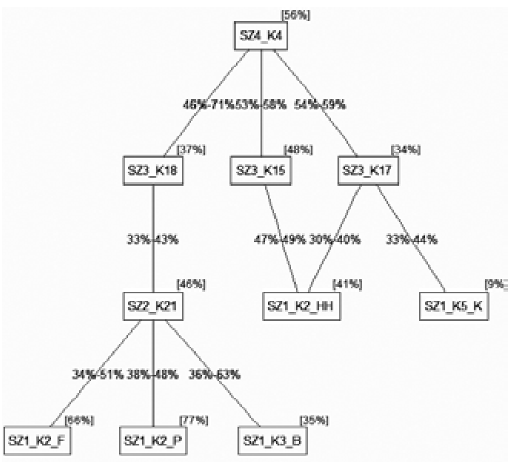

Diagram 8. The Hasse diagram of the second theoretical model representing 10 questions and their prerequisite relations

5. Fitting of the knowledge spaces to observed data

It is an important task to examine the fitting of hypothetical knowledge structure and knowledge structure based on empirical data. So far, we have set up two models based on the theory of knowledge structure for the two groups of questions in our questionnaire, and now we turn to examine how these models fit the data from the questionnaires. The analysis of fitting can be considered our search for the best fitting knowledge structure to our data. When examining the degree of fitting, we need to take into consideration if lucky guesses or careless errors are included in the model or not. We have selected some coefficients in the present study which do not directly include the two above-mentioned distorting factors. Furthermore, the fitting tests applied do not offer significance values, only goodness indices based on the theory of knowledge space, where solely the values of perfect fitting mean the base for comparison. Therefore, we created simulated databases according to the theoretical models, which necessarily give a perfect reflection of the models mentioned, however, these simulations also include the possibility of lucky guesses and careless errors. We expect that the resulting data will not fit significantly better, and the goodness indices will be similar to the ones resulting from the data of the questionnaire. All this would prove that knowledge space theory is well applicable to the analysis of the data concerning the historical questionnaire, however, the limits of the multiple choice tests need to be taken into account.

5.1 Description of the simulation

Simulation helps us in imitating a real phenomenon, mostly with the help of a computer. In our case, this phenomenon is the completion of the worksheet by individuals. The result of the simulation will be the correct or incorrect answers given to the questions, that is either 1 or 0. As the element number of the original sample (number of individuals) was 169, we repeat the simulation 169 times. We conduct two types of simulation: the first one is based on *theoretical model 1*, and the second one on *theoretical model 2*.

The models used in the simulation have two kinds of parameters (in the case of *a b*: *a* covered by *b*, *a* – prerequisite question; *b* – covering question).

- p_0: how probable it is that the individual knows the answer for the covering question if s/he gave an incorrect answer to the prerequisite question. The value of this in the simulation is 0.35. This value – beyond the previously described theoretical consideration – can be considered typical in the sample.
- p_1: how probable it is that the individual knows the answer for the covering question if s/he gave a correct answer to the prerequisite question. The value of this in the simulation is 0.7. This is the upper limit of this type of probability in the sample.

Two basic cases can be differentiated in the relation between prerequisite and covering questions:

Case 1 one covering question is related to one prerequisite question
Case 2 one covering question is related to more prerequisite questions

It is enough to consider these basic cases during simulation, as this allows us to assign either 1 or 0 to all questions, that is, a correct or incorrect solution.

The steps of the simulation are the following:

1. We assign 0 or 1 to the leaf questions (without prerequisite question) of the Hasse diagram (Diagram 7 or 8) at a probability of 0.5.
2. If a covering question has only one prerequisite one (Case 1), then the covering question will take value 1 at a probability of p_0 if the prerequisite question is 0, or it will take value 1 at a probability of p_1 if the prerequisite question is 1.
3. If a covering question has only multiple prerequisite questions (Case 2), then the covering question will take value 1 at a probability of p_0 if there is of value 0 among the prerequisite questions, or it will take value 1 at a probability of p_1 if all the prerequisite question take value 1.

On the basis of the above simulation, we conducted a so-called *modified simulation* as well, which is closer to knowledge space theory. The only modification is that it does not only take into consideration the prerequisite question in step 2 and 3, but also the way that leads there. If there is 0 somewhere there, it will consider the prerequisite question 0, too.

5.2 A brief presentation on the goodness indices of fit

The simplest way to express the goodness of fit of theoretical (hypothetical) models and response patterns is to determine the distance between them. We search for the knowledge state in the model for each response pattern which it is closest to, that is, where the deviation of 0s and 1s is the smallest. If, for example, in the case of a worksheet with 5 questions (*a*-*e*), a student does not know the answer for the first three questions but s/he responds correctly to the last two, his/her response pattern will be 00011. It may occur in a theoretical model that exactly this knowledge state is also present, and the distance is 0 in this case. However, if it is not present, and closest to it we find 10111, then the minimal distance will be 2. It is with the help of a frequency table that we can describe how many times distance 0 (perfect fit), distance 1, distance 2, etc. occur in the case of all the response patterns. From the point of view of the goodness of fit, it is more favourable if high frequency values occur at small distance elements. It is possible to express distance-based fit with a single number, as it is enough to consider the average of the given distances (0, 1, 2, etc.) weighted by frequency numbers. This is called *average distance* (denoted *ddat*), and the lower value is more favourable, of course. The last index related to distance is the Distance Agreement coefficient, which is calculated in the following way: DA=ddat/dpot. The value dpot is actually an average distance where all the possible response patterns are compared to the theoretical model, not the response patterns resulting from the survey. Lower DA values mean better fit.

Another two indices are used to examine the goodness of fit, which are used for examining the validity of precedence relations. In the case of both indices, we take into consideration the response patterns, which confirm or disaffirm the precedence relations in the theoretical models. The value of the so-called *gamma-index* can vary between -1 and +1, where a positive gamma-index value generally means the validity of precedence relations. Its interpretation and calculation is similar to the gamma index introduced by Goodman-Kruskal (Goodman and Kruskal, 1972). The Violational Coefficient index (VC index) expresses the ratio of disaffirming response patterns compared to the total number of precedence relations, thus a lower VC value means the better validity of precedence relations.

The exact description of the above indexes can be found in Appendix 2.

6.3 The analysis of the fitting of models

The indices of fit introduced in the above point are summarised in Table 11. Distance indices are represented in the columns „Frequency tables of distances", „ddat - average distance" and „DA". The validity of precedence relations are expressed by the values in columns "gamma-index" and „VC". In the heading of the last four columns, we indicate with arrows which are more favourable index values (↓ - lower, ↑ - higher) concerning fitting. The first three rows of the table refer to *theoretical model 1*, whereas the last three rows to *theoretical model 2*.

Table 11. Fitting the models to response patterns by 169 students and to data resulting from simulation.

		Frequency tables of distances						ddat - average distance (?)	DA (?)	gamma-index (?)	VC (?)
Distance		0	1	2	3	4	5				
theoretical model 1	Data	3	15	60	45	41	5	2.71	0.85	-0.0012	0.2235
	Simulation	3	18	38	55	50	5	2.86	0.90	0.0595	0.2161
	Modified Simulation	5	12	54	58	38	2	2.70	0.84	0.1870	0.1873
theoretical model 2	Data	7	27	55	59	21	0	2.36	0.84	-0.0190	0.2342
	Simulation	11	22	42	70	24	0	2.44	0.87	0.0479	0.2105
	Modified Simulation	7	28	64	55	15	0	2.25	0.80	0.1930	0.1803

As the table shows, there is hardly any difference between the data resulting from the questionnaire and the ones created by simulation. The tendency is evident, the best results are given by modified simulation – which reflects best knowledge space theory -, but indices in the rows of empirical data do not differ significantly either.

7 SUMMARY

We have examined in our study whether knowledge space theory is applicable in representing historical knowledge, if so, in what way. The domain in question was Lajos Kossuth and the Hungarian War of Independence. We prepared a questionnaire on the topic, and it was filled in by 169 high-school and university students. The basis of our analysis results from this questionnaire and the data on the given responses.

The questionnaire has several stages and included multiple-choice questions. First, the relevance of concepts had to be decided on, then gradually more complex questions had to be answered in three levels, with four possible answers. We set up two models for the two groups of questions, on the basis of the supposed precedence relations between responses, and we tested the fitting of the models to the data in the framework of knowledge space theory. The result we arrived at was that the fitting was almost as good as that of the simulated model, whose only fault is to include the probability of lucky guesses and careless errors.

Our findings offer the possibility to use knowledge space theory in the domain of history when setting up assessment strategies. If the framework of multiple choice tests is retained, the conclusion of this study is that it is worth starting the assessment at the first level, with the relevance of names and concepts, then with their meaning, and it is advisable to move on to next stages in the topic only if appropriate knowledge has been achieved at the first two levels. What exactly is to be considered appropriate knowledge should be further investigated.

Appendix A. Basic Mathematical Concepts

Many of the concepts described in this study operate on sets and relations. Only the fundamental concepts of set theory, which are relevant for explaining knowledge space theory concepts in this study, are explained here. We shall follow the approach taken by Falmagne and Doignon (2011).

- **Sets.** A set is a collection or group of definable *elements* or *members*, which we denote by a capital letter (e.g., Q, A, B, X). We denote an element by a lower-case letter (e.g., x, y, a, b), and state that "*x is an element of X*" with the notation $x \in X$. The expression $Q=\{a, b, c, d\}$ says that the set Q consists of a collection of 4 elements, a through d. A set can be also characterized by the properties of its elements, e.g., $A = \{a : a > 1\}$. The *null set* ($\varnothing$) is an *empty set*, or a set with no elements.
- **Subset, power set.** We say that B is a *subset* of A if all the elements of B are also elements of A. We write it as $B \subseteq A$. The *power set* of any set Q is the set of all possible subsets of Q and is denoted by 2^Q. Every power set of any set Q must contain the set Q itself and the empty set $\varnothing$. The *size* or *cardinal number* of a set Q is denoted by $|Q|$
- **Cartesian product**. The *Cartesian product* of sets A and B, denoted by $A \times B$, is the set $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$. We call (a, b) an *ordered pair* or a *tuple*.
- **Relations.** Let A and B be sets. A *binary relation* R between A and B is a subset of $A \times B$. Given a tuple (a, b) in $A \times B$, we say that a is related to b by R if $(a, b) \in R$ and is denoted by $(X, P)aRb$.
- **Properties of relations**. A relation $R \subseteq X \times X$ is
  reflexive, when $(x, x) \in R$ for all $x \in X$;
  symmetric, when $(x, y) \in R$ implies $(y, x) \in R$ for all $x, y \in X$;
  asymmetric, when $(x, y) \in R$ implies $(y, x) \notin R$ for all $x, y \in X$;
  antisymmetric, if $(x, y) \in R$ and $(y, x) \in R$ implies $x = y$ for all $x, y \in X$;
  transitive, if $(x, y) \in R$ and $(y, z) \in R$ implies $(x, z) \in R$ for all $x, y, z \in X$;
  total, if for all $x, y \in X$, $(x, y) \in R$ or $(y, x) \in R$ (or both).

- **Quasi orders, partial orders, linear orders**. A *quasi order* is a binary relation R over a set X, which is reflexive and transitive. An antisymmetric quasi order is *partial order*. A total, antisymmetric and transitive binary relation over X is *linear order*. A set X with a quasi (partial, total) order P is called a *quasi* (*partially, totally) ordered* set and denoted by (X, P).
- **Covering relation, Hasse diagram.** Let (X, P) be a partially ordered set. We say that x is *covered* by the element y when xPy and there is no $z \in X$ with $xPzPy$. The *covering relation* or *Hasse diagram* of (X, P) contains all the pairs (x, y) with y covering x. When X is finite and small, the Hasse diagram of P can be simply displayed by a *directed graph* drawn according to the following conventions: the elements of X are represented by vertex in the plane, with an line segment or curve (edge) that goes upward from x to y whenever y covers x.

APPENDIX B.
FITTING OF THE KNOWLEDGE SPACES TO OBSERVED DATA

Distance measures and validity coefficients are described in Schrepp, Held & Albert (1999) and Stahl (2008).

B.1. Distances between knowledge space and response patterns

We already gave the definition of response patterns in section 4.2, but here we shall rely on a equivalent variant. We shall represent a *response pattern* by the subset R of Q containing all the questions correctly solved by the subject. There are thus $2^{|Q|}$ possible response patterns.

The distance between a response pattern R and a knowledge state K is defined as the number of elements in the symmetric difference of R and K, that is:

$$dist(R, K) = |(K \setminus R) \cup (R \setminus K)|.$$

The *minimal symmetric distance* of a response pattern R to a knowledge space $\mathcal{K}$ is defined as the distance of R to the nearest knowledge state in $\mathcal{K}$, that is:

$$mdist(R, \mathcal{K}) = \min\{dist(R, K) \mid K \in \mathcal{K}\}.$$

When the response pattern is identical to a knowledge state, the minimal symmetric distance is zero. Positive value indicates a difference between R and $\mathcal{K}$.

The first indicator of fitting is the *distribution of the minimal symmetric distance* over a large number of subjects' response. The second is the *average of minimal symmetric distances*, denoted by *ddat*. The third, the *Distance Agreement Cofficient* (*DA*), compares the average symmetric distance between the knowledge space $\mathcal{K}$ and individuals' response patterns (referred to as *ddat*) to the average symmetric distance between the knowledge space $\mathcal{K}$ and the power set of response patterns 2^Q (referred to as *dpot*). Formally, *DA* is calculated as the ratio of *ddat* and *dpot* (*DA=ddat*/*dpot*).

B.2 The gamma-Index and the Violational Coefficient (VC)

The gamma-index and the violational coefficient calculate validity coefficients for prerequisite relations and knowledge spaces.

The gamma-Index is similar to Goodman and Kruskal's Gamma (Goodman and Kruskal, 1972). It depends on two quantities, the number of pair of problems that are represented by a prerequisite relation (i.e., concordant pairs, N_c), and the number of pair of problems that are not represented by a prerequisite relation (i.e., discordant pairs, N_d). A pair of problems in a response patterns are concordant if the prerequisite problem is correctly solved (1) and the covering problem is failed (0). A pair of problems are discordant if the prerequisite problem is failed (0) and the covering problem is correctly solved (1). All tied pairs are ignored (1-1 and 0-0). Gamma-index is calculated by

$$\gamma\text{-index} = (N_c - N_d) / (N_c + N_d)$$

The Violational Coefficient (VC) also validates prerequisite relations. The number of violations (i.e., the earlier mentioned discordant pairs) against a prerequisite relation are calculated, and defined as

$$VC = \frac{1}{n \cdot (|\leq| - m)} \sum_{i,j} v_{ij}$$

where n denotes the number of subjects, $|\leq|$ refers to the number of pairs in the relation, m denotes the number of problems, and v_{ij} again refers to the number of discordant pairs.

Reference

Abari, K., Máth, J. (2010). A történelmi tudás mérése a tudástér-elmélet segítségével. . In: Münnich, Á., Hunyady, Gy. (szerk.). *A nemzeti emlékezet vizsgálatának pszichológiai szempontjai.* Budapest: ELTE Eötvös Kiadó.

Albert, D., & Lukas, J. (Eds.). (1999). *Knowledge Spaces: Theories, Empirical Research, and Applications.* Mahwah, NJ: Lawrence Erlbaum Associates.

Albert, D., & Held, T. (1999). Component-based Knowledge Spaces in Problem Solving and Inductive Reasoning, In D. Albert & J. Lukas (Eds.), *Knowledge Spaces: Theories, Empirical Research, and Applications* (pp. 15–40). Mahwah, NJ: Lawrence Erlbaum Associates.

Bart, W. M., Krus D. J. (1973). An Ordering-Theoretic Method to Determine Hierarchies Among Items. *Educational and Psychological Measurement*, 33, 291–300.

Falmagne, J., & Doignon, J. (2010). *Learning Spaces: Interdisciplinary Applied Mathematics*. Berlin: Springer–Verlag.

Doignon, J., & Falmagne, J. (1999). *Knowledge Spaces*. Berlin: Springer–Verlag.

Doignon, J. (2001). Knowlede Spaces. In N. J. Smelser & P. B. Baltes (Eds.), *International Encyclopedia of the Social and Behavioral Sciences* (pp. 8148–8151). Amsterdam: Elsevier.

GOODMAN, L. A., KRUSKAL, W. H. (1972) Measures of association for cross classication. *Journal of the American Statistical Association*, 67:415-421.

HELD, T., & KOROSSY, K. (1998). Data-analysis as heuristic for establishing theoretically founded item structures. *Zeitschrift für Psychologie*, 206, 169-188.

KATONA, A., SALLAI, J. (2002). *A történelem tanítása. Tantárgypedagógiai összefoglaló.* Budapest: Nemzeti Tankönyvkiadó.

KIRÁLY, Z. (2010). Magyar történelmi személyek és helyek a középiskolai történelemoktatásban, a kerettantervek és a tankönyvek tükrében. In: Münnich, Á., Hunyady, Gy. (szerk.). *A nemzeti emlékezet vizsgálatának pszichológiai szempontjai.* Budapest: ELTE Eötvös Kiadó.

KOROSSY, K. (1999). Modeling Knowledge as Competence and Performance. In Albert, D., & Lukas, J. (Eds.), *Knowledge Spaces: Theories, Empirical Research, and Applications* (pp. 103-132). Mahwah, NJ: Lawrence Erlbaum Associates.

VAN LEEUWE, J. (1974). Item Tree Analysis. *Nederlands Tijdschrift voor de Psychologie*, 29, 475–484.

R Development Core Team (2009). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0, URL http://www.R-project.org.

SCHREPP, M. (1999). On the Empirical Construction of Implications on Bi-valued Test Items. *Mathematical Social Sciences*, 38(3), 361–375.

SCHREPP, M., Held, T. és Albert, D. (1999). Component-based construction of surmise relations for chess problems, In D. Albert & J. Lukas (Eds.), *Knowledge Spaces: Theories, Empirical Research, and Applications* (pp. 41–66). Mahwah, NJ: Lawrence Erlbaum Associates.

STAHL, C. (2008). *Knowledge Space Theory.*
URL http://cran.r-project.org/web/packages/kst/vignettes/kst.pdf.

SZÁRAY, M. (2009).*Történelem III. - Középiskolák, 11. évfolyam.* Budapest: Nemzeti Tankönyvkiadó.

TÓTH, Z. (2005). A tudásszerkezet és a tudás szerveződésének vizsgálata a tudástér-elmélet alapján. *Magyar Pedagógia, 105. (1)*, 59–82.

ÜNLÜ, A. and SARGIN, A. (2010) DAKS: An R package for data analysis methods in knowledge space theory. *Journal of Statistical Software*, 37(2), 1–31.
URL http://www.jstatsoft.org/v37/i02/.